Multiple Access in Connectionless Networks Using Cooperative Transmission*

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Abstract

The paper calculates the throughput of Carrier Sense Multiple Access (CSMA) assuming that the set of propagation delays among the nodes (the so called "vulnerable times") are very different and shows that considering the worst case propagation time can be largely pessimistic in assessing the performance of these networks. The application of our analysis is a large ad-hoc wireless network of locally interconnected nodes where, to transmit a packet, each node floods the entire network to avoid routing. Assuming that the nodes access the channel using CSMA, collisions occur if the network starts to be flooded by multiple sources which, due to the various delays ("vulnerable time") necessary for the first signal to reach them, can fail to sense that the channel is already in use and that the network is being flooded by other nodes. The paper considers, as flooding strategy, a scheme where the packets are broadcasted using multiple levels of relays at the physical layer, where the nodes progressively echo, on a symbol by symbol basis, what they receive from the near neighbors. In this scheme, called Opportunistic Large Array (OLA), the intermediate nodes that act as relays cannot collide unless they carry information from different sources. OLA also achieves a cooperative wireless advantage compared to other broadcasting techniques that comes from the fact that multi-user interference in OLA is equivalent to active multi-path propagation and thus is a benign source of diversity and signal enhancement.

1 Introduction

Ad hoc networks are of considerable importance to provide networking capabilities in the absence of a fixed infrastructure. Having resilient and efficient networking and multiple access strategies is key to render these type of networks of practical use. The techniques proposed in this paper address both aspects of the problem. We discuss these problems in the next two items.

- **Routing for connectionless networks**
  To establish peer-to-peer connections in a network of locally connected nodes, utilizing intermediate nodes as relays and using multi-hop route allows to save transmission power. Unfortunately, in highly dynamical ad hoc networks, implementing even moderately demanding routing protocols can be completely impractical. In fact, as the network size grows, finding and maintaining routing tables becomes

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computationally very expensive and time/bandwidth consuming. In fact, an alternative to finding the route to the destination is to broadcast the information to all nodes, performing what is called *network flooding*. Since the information floods the entire network, routing is unnecessary and these networks are called *connectionless*.

- **Multiple Access in connectionless networks**
  In connectionless networks, if multiple sources intend to reach their destination and hence, flood the network, we have to solve a multiple access problem that, possibly, extends to the scale of the entire network where not only the sources have to be multiplexed but also all the nodes that act as relays. The complex interactions among the nodes, as the packets move across the network, seem to lead to an intractable problem.

  The problem of multiplexing multiple sources in the ad-hoc wireless networks is, in fact, not trivial. TDMA requires a centralized synchronism signal that is lacking in a distributed network. High node density renders FDMA inefficient. The time-varying nature of different signatures generated by the different users makes the use of CDMA scheme interference limited [1]. All the multiplexing techniques above are affected equally by the absence of a central control that assigns time slots, frequency bands or codes, resolving the contention problem. Hence, in the fully decentralized scenario, it is more reasonable to resort to random access schemes.

In connectionless networks, when two or more sources access a channel simultaneously, there are two types of collisions that can occur as the packets from the sources move forward in the network: **Type 1**) collisions that occur due to lack of coordination of the relay nodes, attempting to access the channel to deliver packets from the same source at the same time (*collisions within a single flood*); **Type 2**) collisions among nodes that are transmitting packets from different sources (*collisions between multiple floods*). Type 2 collisions are illustrated in Fig. 1.

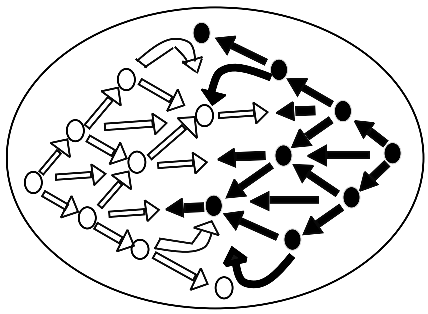


Figure 1: Type 2 Collision: When two nodes broadcast in the overlapping periods.

Our contribution in this paper is twofold: 1) we propose a broadcasting technique that eliminates Type 1 collisions in connectionless networks; 2) we propose to use our broadcasting technique in combination with CSMA and derive analytically what is the network throughput, i.e. the average number of packets that are delivered successfully over the entire network.

The key idea is that, if Type 1 collisions are impossible and the network is flooded, the throughput analysis can be performed with the same analysis tools of a single-hop packet radio network. In the following we will better motivate our broadcasting strategy and choice for CSMA.

1.1 CSMA and Physical Layer Broadcasting

It is well known that CSMA [2] is an efficient random access protocol for single-hop packet radio networks. In a network where the nodes sense the channel to be available to relay the information from a potential source, it is natural to utilize carrier sensing also to regulate the channel access. However, in multi-hop networks, inevitably, sensing the channel gives only the *local* insight on the network status, because the nodes are locally connected and, thus, are only capable of sensing their neighbors. Because of the local connectivity, multi-hop networks suffer from the problem of *hidden* nodes. It is evident that the presence of *hidden* terminals in the network degrades the performance of CSMA [3] and this is a serious problem for networks that are locally interconnected. Many reservation based MAC protocols like MACA [4], MACAW [5], FAMA [6] have been proposed in literature for ad hoc networks. These rely on Request To Send (RTS)/ Clear To Send (CTS) message exchanges during the communication between nodes to reduce the chances of collision. This increases the time required to access the channel and hence the delay. In the literature, the analysis on the network throughput is always local (i.e. based on fully connected network) while the large scale network performance is always obtained by simulations. In this paper, we offer one of the first examples of random access analysis, that is valid on a global scale, for networks that are only locally connected. To address this difficult problem, we limit our study to a broadcast scenario where we assume that the transmitter wants to flood the entire network with its information.

A cooperative form of transmission for large wireless networks (OLA) was recently proposed in [7] to flood the entire wireless network, on a symbol-by-symbol basis; this is in contrast to the standard packet based transmission of multi-hop networks. The main reason to perform the relaying process this way is that the relays do not have to contend for the transmission medium, since multi-user interference in OLA is equivalent to active multi-path propagation and thus is a key source of redundancy, coordination, diversity and signal enhancement. This mechanism, as we anticipated in the previous section, eliminates what we denote as Type 1 collisions. In the broadcast scenario we consider, a terminal is *hidden* if it is not connected [7]. With OLA, the nodes that operate as relays, as will be more clear in Section 2, do not have contention, since their transmission is meant to overlap. In addition, the nodes, even though through multiple levels of relay, can "sense" when the channel is busy as if there is only one hop that separates them from the source and, hence, the problem of *hidden* terminals is avoided, so long as the network is connected.

Two waves of information signals from separate sources may never imping on each other if they move away from the same direction at separate times (c.f. Figure 1). Considering that the relay process occurs on a symbol by symbol basis, we will consider those events as collisions as well in our analysis. Interestingly, this allows to analyze the multi-hop network throughput with tools that are analogous to those used in single-hop packet networks, and provide an analysis that is valid on a global scale. The main difference of the multi-hop scenario is that the propagation delays of the information floods are very diverse, so the standard CSMA analysis, which considers the worst propagation delay, provides a loose lower bound for the performance. It will be shown in Section 4 that the throughput achieved is quite higher than what the above lower bound predicts[1].

[1]Note that our analysis, apart from the multi-hop scenario described above, could also be applied to a long ethernet cable which has significantly different propagation delays among the users.

2 Cooperative Transmission through OLA

In this section, we briefly talk about the cooperative transmission mechanism, OLA, proposed in [7] and point out the steps to calculate the vulnerable durations[2] in the multi-hop network. In the basic OLA scheme, the source or *leader* transmits a pulse with complex envelope $p^{(i)}(t)$ out of an M-ary set of waveforms. Neglecting errors, the resulting signal at the jth receiver is: $r_j(t) = s_j^{(i)}(t) + n_j(t)$, where $n_j(t)$ is the jth receiver AWGN with variance N_0 and $s_j^{(i)}(t)$ is the signature waveform generated by the network when the ith symbol is relayed, which is given by

$$s_j^{(i)}(t) = \sum_{n=1}^{N} A_{j,n}(t) p^{(i)}(t - \tau_{j,n}(t)), \quad i = 0, \ldots, M-1, \tag{1}$$

where $\tau_{j,n}(t)$ is the delay of the link between the jth and the nth node and $A_{j,n}$ is the product of a complex fading coefficient times the transmit power times the average channel gain, e.g. $\propto \ (1 + d_{j,n})^{\beta}$ (log-normal fading), where $d_{j,n}$ is the distance between the jth and the nth node and β is the path loss exponent. With slow channel variations and limited node mobility, we assume that $A_{j,n}$ and $\tau_{j,n}$ are constant over multiple symbol durations. Hence, their time dependence in (1) can be omitted. The data rate of the *leader* is limited by the delay spread of the network signature it generates. To avoid ISI, the upper bound for the effective symbol rate is $R_{s,i} \leq \frac{1}{\triangle\tau_i}$, where $\triangle\tau_i$ is the maximum delay spread of $s_j^{(i)}(t)$ for all j, i.e.

$$\triangle\tau_i = \max_j \sqrt{\frac{\int_{-\infty}^{\infty}(t - \bar{\tau}_{i,j})^2 . |s_j^{(i)}(t)|^2 dt}{\int_{-\infty}^{\infty} |s_j^{(i)}(t)|^2 dt}}, \qquad \bar{\tau}_{i,j} = \frac{\int_{-\infty}^{\infty} t . |s_j^{(i)}(t)|^2 dt}{\int_{-\infty}^{\infty} |s_j^{(i)}(t)|^2 dt}. \tag{2}$$

To guarantee a certain BER performance, the nodes in the vicinity of the leader echo its transmission only after the signal they receive achieves a certain SNR threshold (SNR_{th}). More formally, we define *connectivity/firing* as follows:

Definition 1 *The node j is said to fire when the energy it accumulates, over time, attains a certain predetermined threshold, i.e. $\int_0^{\tau_f(j)} \xi |s_j^{(i)}(v)|^2 dv = SNR_{th} \ \ \forall i$, where $\xi = P/N_0$ for the convenience of notation and P is the transmitted power. The time $\tau_f(j)$ is called the firing instant of the node j, with it being connected if $\tau_f(j)$ is finite.*

The nodes, whose SNR meets the above criterion, aid in the relaying process. In OLA, simultaneous transmissions from multiple leaders are considered the source of contention (Type 2). CSMA analysis relies on the capability of the potential transmitters to sense the state of the channel and to restrain themselves from transmission in the event of channel being sensed busy. This critical time is called the "vulnerable time", which is the time elapsed between the events of the leader initiating the transmission and the next potential source hearing the leader's transmission. This time duration coincides (at most) with the time, the next potential source fires. Hence, the calculation of the vulnerable times calls for finding the firing instants $\tau_f(j)$'s of the nodes, which is done next.

[2]This is time taken by a source terminal to sense the channel. If more than one symbol arrive during this period, the collision is inevitable.

2.1 End to End delay calculation

As discussed above, the calculation of the firing instants requires evaluating the energy received at the node j, which is the integral over the square of the signature waveform, i.e. $\int |s_j^{(i)}(v)|^2 dv$ and therefore, the energy accumulated at time t can be written as

$$E_j^{(i)}(t) = \sum_{n_1,\ n_2=1}^{N} A_{j,n_1} A_{j,n_2} \int_0^t p^{(i)}(v-\tau_{j,n_1}) p^{(i)}(v-\tau_{j,n_2}) dv \qquad (3)$$

For the sake of simplicity, we assume BPSK modulation, i.e. the nodes transmit a pulse from the set $\{p(t), -p(t)\}$ and therefore, dependence on i can be removed. Assuming log-normal fading with P as the transmit power, (3) normalized by N_0 becomes

$$E_j(t) = \xi \sum_{n_1,\ n_2=1}^{j-1} \frac{\int_0^t p(v-\tau_f(n_1)-\frac{d_{j,n_1}}{c})\ p(v-\tau_f(n_2)-\frac{d_{j,n_2}}{c}) dv}{(1+d_{j,n_1})^\beta (1+d_{j,n_2})^\beta} \qquad (4)$$

where we have assumed that the nodes are enumerated in order of their firing instants, i.e. all nodes up to $j-1$ have fired. c is the speed of light in vaccum.

The integral in (4) can be solved exactly if the nature of $p(t)$ is known. To enable thorough treatment of the problem at hand, we consider $p(t)$ to be a rectangular pulse of duration T_p and the integrand then consists of the product of two rectangular pulses starting at $A_j(n_1) \triangleq \tau_f(n_1) + \frac{d_{j,n_1}}{c}$ and $A_j(n_2) \triangleq \tau_f(n_2) + \frac{d_{j,n_2}}{c}$, respectively. Hence,

$$E_j(t) = \xi \sum_{n_1,n_2=1}^{j-1} \frac{[\min\{t, \min\{A_j(n_1)+T_p, A_j(n_2)+T_p\}\} - \max\{A_j(n_1), A_j(n_2)\}]^+}{(1+d_{j,n_1})^\beta (1+d_{j,n_2})^\beta} \qquad (5)$$

where $[a]^+$ denotes $\max\{0, a\}$.

In order to illustrate the steps for the calculation of firing instants, we consider simplified geometry as shown in Figure 2, a linear grid that extends in one direction[3], where d is the inter-node distance. We will try to bound the firing instants from above by as-

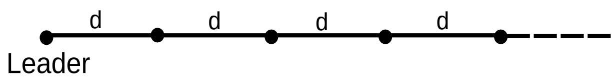


Figure 2: Unicast case.

suming, in (5), that $\tau_f(j) \geq \max_{1\leq\ n_1 \leq\ j-1,\ 1\leq\ n_2 \leq\ j-1} [\max\{A_j(n_1), A_j(n_2)\}]$. This gives us a reasonably tight upper bound for firing instant of node j. The energy expression is

$$E_j(t) = \xi \sum_{n_1,n_2=1}^{j-1} \frac{t - \max\{A_j(n_1), A_j(n_2)\}}{(1+(j-n_1)d)^\beta (1+(j-n_2)d)^\beta}. \qquad (6)$$

The threshold is chosen such that the immediate neighbor of the leader is connected to trigger the avalanche; thus $\xi/(1+d)^{2\beta} \geq \mathrm{SNR_{th}}$ must be satisfied (c.f. Figure 2). From

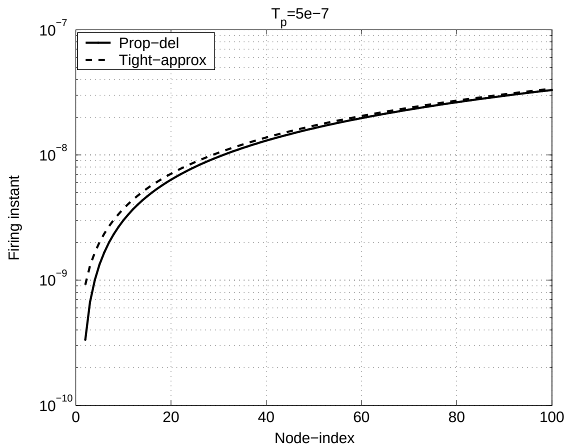
[3]The motivation behind this network was the prior knowledge of firing order, which is necessary as mentioned before.

the definition of the firing instant[4], (6) results in the following:

$$\tau_f(j) \;=\; \frac{\frac{\mathrm{SNR_{th}}}{\xi} + \sum_{n_1,n_2=1}^{j-1} \frac{\max\{A_j(n_1),A_j(n_2)\}}{(1+(j-n_1)d)^\beta(1+(j-n_2)d)^\beta}}{\sum_{n_1,n_2=1}^{j-1} \frac{1}{(1+(j-n_1)d)^\beta(1+(j-n_2)d)^\beta}}. \tag{7}$$

We plot firing instants of all the nodes in a 100-node unicast network in Figure 3, calculated using the above expression. The bound is quite tight and approaches the natural lower bound, i.e. the propagation delay[5] as the network grows. Of course, we have neglected any additional delay necessary to perform the decision at the node.

To show that the firing instants are in the order of propagation delay for any random geometry, we placed 100 nodes uniformly in a square of side 10m and compared the firing instants of the nodes with the propagation delay and obtained the Figure 3(b). With this result in mind, we will use the propagation delays as the "vulnerable times" in our simulations, while assessing the performance of our system with multiple sources using CSMA in Section 5.



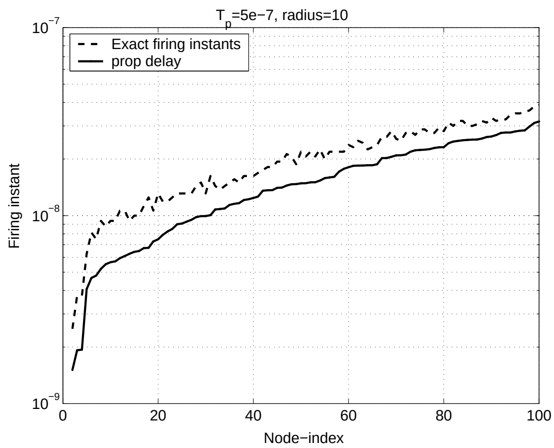


Figure 3: (a) Firing instants for the unicast network, (b) Random network with $\xi = 10^2$, $\beta = 2$ and $\mathrm{SNR_{th}} = 10\mathrm{dB}$.

3 Notation and Assumptions

Consider a network with large number of nodes trying to broadcast their packets. We will borrow most of our notation from [2]. Assume that packets arrive for transmission at *each* of the M source nodes according to a Poisson process of rate λ packets/sec. Each of the source node is assumed to have at most one packet waiting to be transmitted at any time, i.e. all the queued packets are discarded. A positive acknowledgement scheme is used to inform the nodes of success/failure[6]. Various protocols proposed in the random access theory differ mainly in action taken by the source nodes after detecting a collision.

In the next section, we evaluate the performance measure of the non-persistent CSMA protocol, based on the seminal work by Kleinrock [2], with different vulnerable durations of all the users. It was seen in [2] that lower bounds on the performance is obtained by choosing the vulnerable duration to be the worst propagation delay. However, this gives a pessimistic bound on the throughput, which is much higher if different vulnerable

[4] $E_j(\tau_f(j)) = \mathrm{SNR_{th}}$.

[5] the time necessary if the signals were travelling at the speed of light

[6] If a source terminal doesn't receive any ack within a certain time-out duration, it assumes collision.

durations are assumed. This assumption is less critical when the network is connected with one RF hop, but is much more pertinent in the multi-hop scenario we consider.

The traffic offered to the channel from each of the source nodes consists of previously collided packets apart from the ones that are newly generated, which we denote by G_i. Note that $G_i = \lambda + q_r p_i$, where q_r is the retransmission probability, that depends on the retransmission delay distribution, and p_i is the probability of the ith source terminal being backlogged. It is also assumed that the average retransmission delay is large compared to the packet transmission duration, T and that the inter-arrival times of the process defined by the starting times of the new and the backlogged packets are independent and exponentially distributed. Note that the assumption that the traffic is Poisson is an approximation that allows us to analyze this difficult scenario. We normalize the time axis with respect to T and denote the normalized vulnerable duration for ith source node, when node j is transmitting, by $a_{ji} = \tau_{ji}/T$, where τ_{ji} is the propagation delay between the nodes i and j.

4 Throughput analysis

Theorem: The throughput, S, of the channel described above, in terms of a_{ji}'s and G_i's is given by $S = \frac{\bar{U}}{\bar{B}+\bar{I}}$, where $\bar{U}$ is the fraction of the busy period used for successful transmission on an average, $\bar{I}$ is the average duration of the idle period and $\bar{B}$ is the average busy period with

$$\bar{U} = \sum_{j=1}^{M} \left(\frac{G_j}{\sum_{k=1}^{M} G_k} \right) \left(\prod_{i=1, i \neq j}^{M} e^{-a_{ji} G_i} \right), \quad \bar{I} = \left(\sum_{k=1}^{M} G_k \right)^{-1} \quad \text{and} \tag{8}$$

$$\bar{B} = \sum_{j=1}^{M} \left\{ (1 + \bar{Y}_j + \alpha_j) \left(\frac{G_j}{\sum_{k=1}^{M} G_k} \right) \right\} \quad \text{with} \tag{9}$$

$$\begin{aligned} \bar{Y}_j \ = & \sum_{i=1,\ i \neq j}^{M} \left[\prod_{k=1,\ k \neq i,j}^{M} e^{-G_k a_{jk}} \left\{ a_{ji} e^{G_k a_{ji}} - \frac{(e^{G_k a_{ji}} - 1)}{G_k} \right\} \right] \times \\ & \left[1 + \sum_{k=1}^{M} (-1)^k \sum_{S \subseteq \{1, \ldots M\}:\ |S| = k} \left(\frac{G_i}{G_i + \sum_{l \in S,\ l \neq i,j} G_l} \right) \right] \quad \text{and} \end{aligned} \tag{10}$$

$$\alpha_j = \max_m \left\{ \sum_{i=1, i \neq j}^{M} a_{im} \left[1 + \sum_{k=1}^{M} (-1)^k \sum_{S \subseteq \{1, \ldots M\}:\ |S| = k} \left(\frac{G_i}{G_i + \sum_{l \in S,\ l \neq i,j} G_l} \right) \right] \right\} \tag{11}$$

Proof: Refer to Appendix A □

5 Analysis for OLA and Numerical Results

In this section, we use the multi-hop broadcast model, described in Section 2, to show the throughput gains achieved using our analysis. Denoting the maximum delay spread due to the lth leader by $\triangle\tau_i^{(l)}$, let R_s be rate at which the symbols are generated at each of the leader terminals, where $\frac{1}{R_s} = \max_l \ \gamma \ \triangle\tau_i^{(l)}$, and $\triangle\tau_i^{(l)}$ is obtained from (2) and γ is a constant chosen to satisfy the ISI constraint. In our scenario, M nodes act as leaders and broadcast over a common wireless channel in an N-node network. Remaining $N-M$ nodes will form the medium of transmission. We considered the packet to be 1000 symbols long in the simulations, i.e. $T = \frac{1000}{R_s}$. If the leaders are allowed to transmit freely, their symbols are bound to collide (Type 2) if their transmission periods overlap (see Figure 1). The vulnerable durations were taken as propagation delays in the simulations, since they were shown to be in the same order in Section 2. As mentioned in the Section 1, the propagation delays between the nodes in our network are quite different, hence, it is worthwhile to examine the throughput of our network as in Section 4.

We plot the throughput obtained against the number of source nodes M along with the throughput obtained by simulating the non-persistent CSMA protocol (see Figure 4(a)). Also shown is the curve for analysis in [2] where fixed vulnerable duration is assumed (solid with circles). First of all, as expected, the throughput is limited by the number of leaders in the network trying to transmit simultaneously. It is seen that we can achieve a greater throughput in our network by considering true values of vulnerable durations as opposed to [2]. The increment in the throughput achieved by our analysis is significant for the networks with non-negligible difference in the vulnerable durations.

In Figure 4(b), the network throughput is plotted as a function of the aggregate offered traffic, $\sum G_k$. The solid curve with triangles shows the throughput obtained in [2]. The upper dashed and solid curves correspond to the analysis and the simulation, respectively. It can be observed that our analysis gives increased throughput as opposed to the case with fixed vulnerable duration.

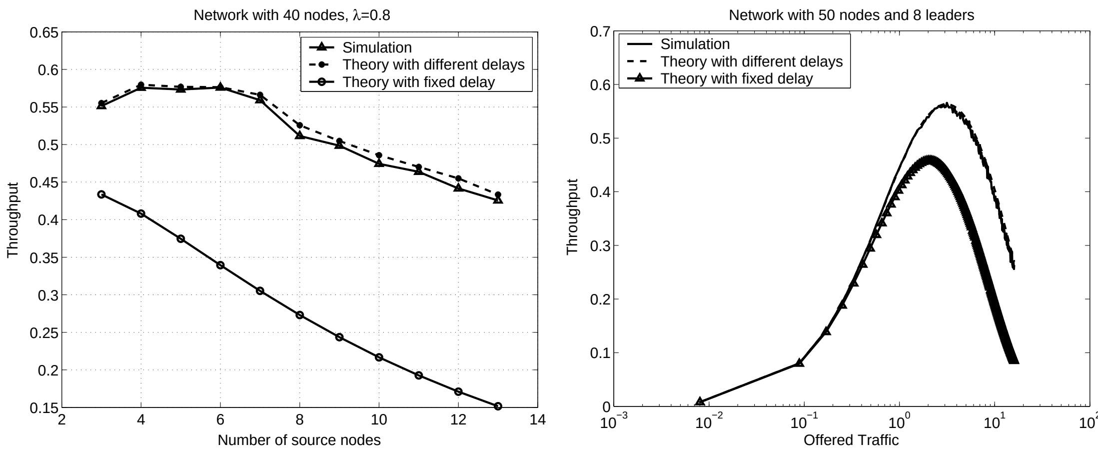


Figure 4: Throughput in non-persistent CSMA with $q_r = 0.1$

6 Conclusions

We considered the problem of multiaccess in a large ad hoc wireless network that utilized as flooding strategy OLA, thereby eliminating routing and multiple access overheads. In OLA there is no contention among the relaying nodes and hence, our CSMA analysis focused only on the contention among potential transmitters (Type 2). It was shown that loose bound on throughput is obtained by considering the worst case propagation delay for the networks with significantly different set of propagation times. Our analysis shows that the throughput achieved is quite higher than what is predicted by this lower bound. We also analyzed the behavior of the network throughput as more and more nodes are tossed over the fixed area. It was observed by simulations that the network throughput was almost insensitive to the size of the network. Hence, we infer that the per-node throughput vanishes as $O(\frac{1}{N})$. This behavior is reasonable considering the fact that we are not routing the packets, rather broadcasting them to all other nodes, unlike[1].

A Proof of the Theorem

Let t be the time instant at which the channel is sensed idle by jth source node and such that no other packet is in the process of transmission. The transmission is successful if, for any other source node $i = 1, \ldots, M,\ i \neq j$, no packets arrive in the interval $(t, t + a_{ji})$

$$\begin{aligned}\bar{U} &= \sum_{j=1}^{M} \Pr \left\{ \begin{array}{c} j\text{th node senses the} \\ \text{channel idle at time } t \end{array} \right\} \times \prod_{i=1, i\neq j}^{M} \Pr \left\{\text{no arrival in } (t, t + a_{ji})\right\} \\ &= \sum_{j=1}^{M} \left(\frac{G_j}{\sum_{k=1}^{M} G_k} \right) \left(\prod_{i=1, i\neq j}^{M} e^{-a_{ji} G_i} \right). \qquad (12)\end{aligned}$$

The first expression in the second line is the probability that node j is the first one to sense the channel idle. This is $\Pr\ \{j = \arg\min_j X_j\}, j = 1, \ldots, M$, where $X_1, \ldots, X_M$, denote the inter-arrival times of the independent point Poisson process, which are exponentially distributed with mean $1/G_j,\ j = 1, \ldots, M$.

The average idle duration, $\bar{I}$, can be found as follows:

$$\bar{I} = \sum_{j=1}^{M} \Pr \left\{ j\text{th node senses the channel idle at time } t \right\} \times \left(\sum_{k=1}^{M} G_k \right)^{-1} = \left(\sum_{k=1}^{M} G_k \right)^{-1} \qquad (13)$$

Now it remains to find $\bar{B}$. Again we condition the analysis on the fact that the node j is the first one to sense the channel idle at time instant t. Let $t + Y_k$ be the arrival time of the packet of the node k in the interval $(t, t + a_{jk})$. Refer to Figure 5. Pick

$$i' = \arg\max_{i'} Y_{i'} \qquad (14)$$

such that $Y_k < a_{jk}$. The transmission of all the packets arriving between t and $t + Y_{i'}$ will

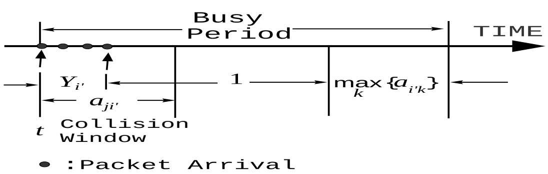


Figure 5: Busy period for non-persistent CSMA protocol.

be completed at $t + Y_{i'} + 1$ and the channel will be sensed unused at most after the time $t + Y_{i'} + 1 + \max_m \{a_{i'm}\}$, i.e. maximum vulnerable duration with respect to the user i' after the completion of transmission. The channel will be busy in $(t, t + Y_{i'} + 1 + \max_m \{a_{i'm}\})$. To calculate $\bar{B}$, we should first average over i' and then over j. The average over i' yields $\bar{b}_j = 1 + \bar{Y}_j + \alpha_j$, where $\bar{Y}_j$ and α_j can be calculated as follows. The distribution function of $Y_{i'}$, given that i' is the solution to (14), is

$$\begin{aligned}\Pr\ \{Y_{i'} \leq y\} &= \Pr\ \{\text{no arrival in } (t + y, t + a_{jk})\ \ \forall\ k \neq i', j\} \\ F_{Y_{i'}}(y) &= \prod_{k=1,\ k\neq i', j}^{M} e^{-G_k (a_{jk} - y)}\end{aligned}$$

$\bar{Y}_{i'}$, conditioned on (14), is

$$E[Y_{i'}|\ i'] = \prod_{k=1,\ k\neq i',j}^{M} G_k e^{-G_k a_{jk}} \int_0^{a_{ji'}} y e^{G_k y} dy = \prod_{k=1,\ k\neq i',j}^{M} e^{-G_k a_{jk}} \left\{ a_{ji'} e^{G_k a_{ji'}} - \frac{(e^{G_k a_{ji'}} - 1)}{G_k} \right\} \quad (15)$$

Computation of $\bar{Y}_j$ requires the following probability

$$\begin{aligned} \Pr\{i' \text{ is the solution to (14)}\} &= \Pr\{i' = \arg\max_{i'\neq j} X_{i'}\} \\ &= 1 + \sum_{k=1}^{M} (-1)^k \sum_{S\subseteq\{1,\ldots M\}:\ |S|=k} \left(\frac{G_{i'}}{G_{i'} + \sum_{l\in S,\ l\neq i',j} G_l} \right) \equiv \Pi_{i'} \end{aligned}$$

where $X_1, \ldots, X_M$, denote the inter-arrival times of the independent point Poisson process, which are exponentially distributed with mean $1/G_i,\ i = 1, \ldots, M$. $\bar{Y}_j$ can now easily be computed as

$$\bar{Y}_j = \sum_{i'=1,\ i'\neq j}^{M} \left[\prod_{k=1,\ k\neq i',j}^{M} e^{-G_k a_{jk}} \left\{ a_{ji'} e^{G_k a_{ji'}} - \frac{(e^{G_k a_{ji'}} - 1)}{G_k} \right\} \right] \Pi_{i'} \quad (16)$$

Now, α_j is obtained by averaging $\max_m\{a_{i'm}\}$ over i', i.e.

$$\alpha_j = \sum_{i'=1,\ i'\neq j}^{M} \max_m \{a_{i'm}\ |i'\} \Pr\{i' = \arg\max_{k\neq j} X_k\} = \max_m \left\{ \sum_{i'=1, i'\neq j}^{M} a_{i'm}\ \Pi_{i'} \right\}$$

Using (16) and (17), $\bar{B}$ can be obtained by averaging $\bar{b}_j$ over j to yield (9).

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