AUTOFOCUSBNG TECHNIQUES FOR SAR IMAGING BASED ON THE MULTILAG HIGH ORDER AMBIGUITY FUNCTION

A. Porchia, S. Barbarossa, A. Scaglione

Infocom Dpt., Univ. of Rome “La Sapienza”
Via Eudossiana 18, 00184 Roma (ITALY)
e-mail: sergio@infocom.ing.uniroma1.it

Georgios B. Giannakis

Dept. of Electrical Engr., Univ. of Virginia
Charlottesville, VA 22903-2442 (USA)
e-mail: georgios@virginia.edu

ABSTRACT

Autofoocusing of SAR images is addressed using a polynomial model for the instantaneous phase shift induced by the relative radar/scene motion. The resulting method is capable of discriminating moving target echoes from the stationary background by exploiting the different phase modulation due to the different motion laws. The algorithm is applied to real SAR images and to computer-generated images obtained by superimposing a simulated echo from a pointlike moving target to a real SAR image.

1. INTRODUCTION

Synthetic Aperture Radar (SAR) produces high resolution microwave images: the range resolution is \( c/2B \), where \( c \) is the speed of light and \( B \) is the bandwidth of the transmitted signal (typically a chirp signal); the cross-range resolution is approximately \( \lambda/2\Delta\theta \), where \( \lambda \) is the transmission wavelength and \( \Delta\theta \) is the angle under which each point on the scene is observed from the radar [4]. The high cross-range resolution is obtained by coherently integrating the echoes received from each point of the observed scene during the observation interval. The coherent integration allows synthesis of an aperture much greater than the real aperture. A correct coherent integration however requires an accurate knowledge of the relative motion between the radar and the observed scene. On-board instrumentation provides the data relative to the position of the aircraft carrying the radar, but the accuracy with which these data are delivered by the instrumentation is much lower than that necessary for an accurate coherent integration. As a consequence, it is necessary to estimate the relative motion directly from the received data. Conventional autofocusng techniques assume a quadratic phase law [4]. However, the requirement for higher cross-range resolution calls for longer observation intervals (longer \( \Delta\theta \)) so that the quadratic assumption is no longer valid. More recently, a quite effective algorithm, called Phase Gradient Autofoocusing (PGA) algorithm [8], was proposed to deal with generic instantaneous phases. PGA estimates the instantaneous frequency on some selected range cells; it then averages the results over several range cells to reduce the estimation errors. The method is not parametric and is valid for a stationary background, but not for a moving target. In this work we propose a parametric method for estimating the instantaneous phase based on a polynomial model of the phase shift. The proposed method is also able to detect and estimate the parameters of moving targets. The paper is organized as follows: Section 2 provides a model for the SAR echo; Section 3 reviews a method for estimating the parameters of multicomponent polynomial phase signals. We will then show some examples of application to real SAR images as well as to real images containing synthetic moving targets.

2. SAR ECHO MODELING

Denoting by \( r(t) \) the vector indicating the radar position at time \( t \) and by \( r'_k(t) \) the vector indicating the position of the generic \( k \)-th scatterer on the ground, the time varying distance between the radar and the \( k \)-th scatterer is:

\[
d_k(t) = |r(t) - r'_k(t)| \approx r(t) - r_{k_1}(t) + \frac{r^2_k(t)}{2r(t)} = r(t) + q_k(t),
\]

where \( r(t) \) is the modulus of \( r(t) \), and \( r_{k_1}(t) \) and \( r_k(t) \) are the projections of \( r'_k(t) \) along directions parallel and perpendicular to the vector \( r(t) \), respectively. The approximation is valid whenever the dimension of the observed scene is much smaller than the radar/scene distance. Transmitting a sinusoidal signal \( s(t) = e^{j2\pi ft} \), the echo \( e(t) \) from \( K \) scatterers located at positions \( r'_k(t) \) is:
\[ e(t) = \sum_{k=1}^{K} A_k \delta(t - 2d_k(t)/c) \]
\[ = \sum_{k=1}^{K} A_k e^{j2\pi f_0 t - j4\pi d_k(t)/\lambda} \]
\[ \approx e^{j2\pi f_0 t} e^{-j4\pi \tau(t)/\lambda} \sum_{k=1}^{K} A_k e^{-j4\pi q_k(t)/\lambda} \]
\[ = e^{j2\pi f_0 t} e^{-j\psi(t)} \sum_{k=1}^{K} A_k e^{-j\phi_k(t)}. \] (3)

If a chirp signal is used instead, a similar expression can be written for each frequency component present in the transmitted chirp. The distances are certainly continuous functions of time, therefore they can be well approximated by finite order polynomials, within a finite observation interval, according to Weierstrass' theorem. In general, a good approximation of \( \psi(t) \) in (3) requires a high order polynomial, especially for long observation intervals, whereas the functions \( \phi_k(t) \) can be well approximated by low order polynomials (usually first order). This second approximation is quite good if the dimension of the observed scene is much smaller than the distance between the radar and the observed scene. Therefore, assuming an \( M \)-th order polynomial model for \( \psi(t) \) and a linear model for \( \phi_k(t) \), the received signal from each range cell, after demodulation, is the following:

\[ z(t) = e^{-j} \sum_{m=0}^{M} a_m t^m/m! \sum_{k=1}^{K} A_k e^{-j2\pi f_0 t} + w(t), \] (4)

where \( w(t) \) is additive noise. Conventional SAR processing removes the common phase term \( \psi(t) \) and then applies an inverse Fourier transform to estimate the amplitudes of each component. The SAR image is then obtained by extracting the modulus of the FFT output, for each range cell. The practical problem is that \( \psi(t) \) has to be estimated. Conventional autofocus algorithms assume that \( \psi(t) \) is a low order polynomial (usually second order polynomial). However, in more recent applications requiring higher resolutions, and then longer observation intervals, the low order assumption is no longer valid. In this paper, we propose the use of a generic order polynomial for \( \psi(t) \). The observed signal is then given by the superposition of \( K \) polynomial-phase signals (PPS).

3. MULTICOMPONENT PPS ESTIMATION

The model in (3) can be used for a stationary scene and for targets moving on the ground. In general, if \( K \) scatterers belong to objects moving with different velocities, the phase functions \( \phi_k(t) \) are not related to each other; conversely, if the scatterers belong to a rigid body, the motions of each scatterer are subject to the constraints imposed by the rigid body. With reference to (4), in the case of stationary scenes, we have PPS components having the same high order coefficients, whereas in the case of moving targets observed against a stationary background the PPS components in general have different phase coefficients. A polynomial model for SAR signals, with the consequent parameter estimation, was already proposed in [2]. However, the method proposed in [2] required the presence of one prominent scatterer. In this work we generalize that approach thus allowing for cases where there are more dominant scatterers in the same range cell. Given a signal \( s(t) \), we define the associated Multi-Lag High order Instantaneous Moment (ML-HIM) by the following iterative rule:

\[ s_1(t) = s(t) \]
\[ s_2(t; \tau_1) = s_1(t + \tau_1)s_1^*(t - \tau_1), \ldots, \]
\[ s_M(t; \tau_1, \ldots, \tau_{M-1}) = s_{M-1}(t + \tau_{M-1}; \tau_1, \ldots, \tau_{M-2}) \times s_{M-1}(t - \tau_{M-1}; \tau_1, \ldots, \tau_{M-2}). \] (5)

The Multi-Lag High order Ambiguity Function (ML-HAF) is then defined as the Fourier Transform, with respect to \( t \), of the ML-HIM:

\[ s_M(t; \tau_1, \ldots, \tau_{M-1}) = \sum_{t} s_M(t; \tau_1, \ldots, \tau_{M-1}) e^{-j2\pi f t}. \] (6)

This function is a generalization of the High order Ambiguity Function (HAF) defined in [7], which is a particular case of the ML-HAF corresponding to lags \( \tau_k \) all equal to each other. The main property of the ML-HIM is that, if \( s(t) \) is a monocomponent PPS of order \( M \), its \( M \)-th order ML-HIM is a complex exponential with frequency: \( 2^{M-1} \prod_{k=1}^{M-1} \tau_k a_M \). Estimating the polynomial coefficients is thus recast to peak-peaking the ML-HAF. Each time a coefficient is estimated, its contribution is removed from the observed signal by multiplication with a reference signal \( e^{-j2\pi M t^{M}/M} \). Once the highest order coefficient has been estimated, the procedure is repeated \( M - 1 \) times to estimate all the phase coefficients, from the highest order one up to the first order. This algorithm was proposed in [5], [6], [1]. In the presence of multicomponent signals, however, this method presents some shortcomings which considerably affect its reliability. In fact, as proved in [3], if the PPS components share the same highest order coefficients, the HAF exhibits undesired peaks which make the estimation ambiguous. Since, according to (4), this is the situation arising in the SAR case, it is
necessary to remove the ambiguity due to the spurious peaks. The ambiguity can be removed by exploiting the redundancy introduced with the multilag definition given in [1], leading to the algorithm proposed in the companion paper [3]. The algorithm starts with the so-called Multiplicative ML-HAF (MML-HAF) defined as the product of $L$ frequency-scaled ML-HAFs obtained using different sets of lags $\tau_i^l$, for $l = 1, \ldots, L$:

$$
S_M^L(f; \tau_1^l, \ldots, \tau_{M-1}^l) = \prod_{i=1}^L S_M(\lambda_i f; \tau_1^i, \ldots, \tau_{M-1}^i),
$$

where the scaling constants $\lambda_i = \prod_{k=1}^{M-1} \tau_k^l / \prod_{k=1}^{M-1} \tau_k^i$ are chosen so that the auto-terms are aligned, whereas the cross-terms are not. The theoretical performance analysis of the MML-HAF is provided in [3].

4. APPLICATION EXAMPLES

In this section we provide examples of application of the proposed algorithm to real SAR images as well to real stationary scenes with superimposed synthetic moving targets.

4.1. Imaging of stationary scenes

Fig. 1 shows a defocused SAR image. Defocusing is due to a cubic phase error introduced for the purpose of testing the proposed algorithm. The estimation algorithm selects one range cell (a row of the image), before the Fourier transform in cross-range is used to form the SAR image. The corresponding HAF is depicted in Fig. 2. Due to the presence of multiple scatterers, the HAF does not exhibit only one peak, as expected in the ideal case. Conversely, the ML-HAF obtained by the multiplication of six HAFs having different sets of lags, shows a clearly isolated peak (see Fig. 3). The cubic phase term is thus compensated and the procedure is applied next to estimate and compensate for the second order phase coefficient. The estimates are then used to autofocus the overall image. The final image is shown in Fig. 4.

![Figure 2: HAF of a row – stationary scene.](image)

![Figure 3: MML-HAF of a row – stationary scene.](image)

4.2. Imaging of targets moving on the ground

Having no access to images containing moving targets, we superimposed to the real SAR image shown in Fig. 4 a signal simulating the presence of a moving target. We neglect shadowing effects due to the moving target. We also suppose that the target motion does not produce an appreciable range migration, so that the echo is confined within one range cell. The HAF of the sequence containing the moving target echo is not meaningful due to the interactions among all the components. However, the MML-HAF, shown in Fig. 5, allows us to distinguish two peaks quite clearly, one due to the stationary scene (all the scatterers belonging to the background contribute to the same peak because their highest order phase coefficient is the same) and the other due to the target. Since the moving target echo occupies only one range cell, comparing the MML-HAFs obtained over different range cells it is possible
to detect the presence of moving targets. On the row where a moving target is detected, we can apply two focusing procedures: the estimation based on one peak produces an image focused with respect to the ground, leaving the moving target defocused, whereas the opposite happens for the estimation based on the other peak. As an example, Fig. 6 shows the row where the moving target has been detected, focused with respect to the moving target.

Figure 5: MML-HAF of a row – stationary scene plus moving target.

5. CONCLUSION

In this work a new autofocusing procedure has been proposed, based on the multiplicative multilag high order ambiguity function, valid for stationary scenes as well as for moving targets. The method has been applied to real SAR images with superimposed pointlike moving targets. The generalization to extended moving targets, exploiting the constraints of a rigid body motion in the estimation procedure, is currently under analysis.

6. REFERENCES


